Cohesion induced by a rotating magnetic field in a granular material

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We report experiments on a magnetic cohesive granular material made of steel spheres in the millimeter range. A magnetic field magnetizes the spheres, so that an interaction force between grains appears. A rotating magnetic field is applied parallel to plane of the quasi-two-dimensional cell containing the spheres so that only the time averaged force between two particles will be considered. Both maximum angle of stability and angles of repose are measured. The maximum angle of stability is found to depend linearly on the interaction force. Another noticeable feature is the lack of dependence of the maximum angle of stability on the initial height of the heap. We show that the angle of repose is less sensitive to the magnetic interaction force than the maximum of stability. At last, we discuss the importance of using a rotating field rather than a constant one. In particular, we report some measurements of both the maximum angle of stability and the angle of repose in constant field, which show a strong dependence of the angles of avalanche on the direction of the field.

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I. INTRODUCTION

Granular materials are present in various contexts including industrial processes, food technology, the pharmaceutical industry, or geophysics. At first, in the 1980's, physicists' attention focused on dry granular materials where, if the grains are sufficiently large, they interact only through contact forces. The mechanical behavior of a granular material is characterized by numerous variables among which the maximum angle of stability (MAS) and the angle of repose are of primordial importance. The MAS is measured by slowly tilting a container filled with the granular material, thereby increasing the angle between the top free surface of the grains and a horizontal plane. When this angle reaches the MAS, an avalanche of grains occurs and the angle of the pile relaxes to a lower angle which is the angle of repose. There exist several methods to measure the angle of repose of a granular material which do not necessarily give exactly the same results. One can pour the grains on a horizontal plane and measure the angle between the top surface of the heap and the horizontal plane. Another method consists of filling a flat bottomed box with the granular material and allowing the grains to flow out through a hole in the bottom of the container, until an equilibrium is reached. The top surface of the grains is no longer horizontal but makes an angle (the angle of repose) with the horizontal plane. In this paper we will use this so-called draining crater method.

The MAS and the angle of repose has been shown to depend drastically on the roughness of the surface of the grains. They are also extremely sensitive to any other interaction forces between grains and will increase as the cohesion between grains is enhanced. The most famous and common example of such interaction force is the adhesive force induced by moisture through capillarity in granular media. It has been observed and measured that even a small amount of liquid added to a granular medium drastically changes its mechanical properties [1–7]. But these changes are not completely understood. In particular, the increase of the MAS cannot be explained by considering simple liquid bridges between ideally smooth particles. Many other questions remain unanswered such as the consequences of the plastic deformation of the particles or the effects of humidity induced ageing [4–7]. Beside the difficulties of the theoretical approaches, the experiments are rather tricky. It is, for instance, difficult to control exactly the degree of humidity in the granular heap and to be sure that it is uniform over the whole volume [3].

Another example of cohesive granular materials is the fine dry cohesive powders [8,9]. Since the surface forces, usually van der Waals forces, scale with the diameter of the particles while the gravitational forces depend on the volume of the particles, reducing the size of the particles is a way to increase the ratio of the cohesion to the weight. Nevertheless, the behavior of such fine powders is made rather complicated by the no longer negligible interaction between the particles and the surrounding gas during the settling of the grains.

Forsyth *et al.* [10] have proposed another method to modify the interparticle forces in a granular media, by applying a vertical magnetic field on a pile of iron particles. The degree of cohesion of this granular media is thus readily tuned by changing the magnetic field strength. They obtained interesting results concerning the dependence of both static and dynamic angles of repose of the pile on the ratio of the magnetic force to the particle weight. Concerning the static experiment, they measure the angle of repose for different values of the constant magnetic field. The force between two particles inside the heap is evaluated by measuring for the same magnetic field the force between an isolated pair of touching spheres aligned in the field direction. The main result they obtain is that the angle of repose increases linearly with the interparticle force. It should be noted, however, that the interactions between the iron particles are highly anisotropic. For instance, the interaction between two adjacent particles is attractive and maximum when the particles are aligned along the field direction while the same adjacent particles repel each other if they are in a plane perpendicular to the field direction. Consequently, the angle of avalanche of

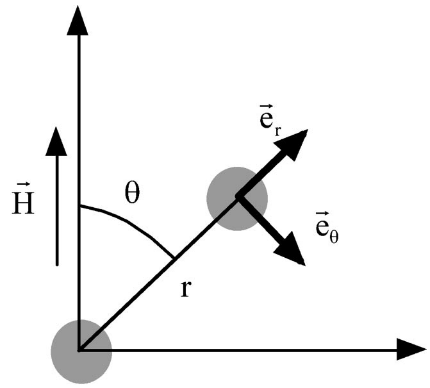

FIG. 1. Two particles in a magnetic field $\vec{H}$. The force in Eq. (2) is exerted on the particle lying at the origin.

such a magnetic granular material should depend on the direction of the field.

In this paper, we present a study of such cohesive magnetic material where, in order to limit the anisotropy of the interaction between grains, we apply a rotating magnetic field. The iron beads are contained in a quasi-two-dimensional (quasi-2D) box (five bead diameters in width), and a rotating magnetic field is applied parallel to the plane of the cell [plane (x,z) in Fig. 6].

In Sec. II, we briefly recall the expression of the magnetic force between two magnetic particles and we present the very simple model used to predict the dependence of the MAS on the intensity of the magnetic field. In Sec. III, we present the experimental device and the main characteristics of the particles. Sec. IV is devoted to the experimental results dealing with the dependence of the MAS and the static angle of repose on the magnetic field strength and the role played by cohesion will be discussed.

II. COHESION INDUCED BY A ROTATING MAGNETIC FIELD

A. The magnetic interaction between two spheres

When a ferromagnetic particle is placed into a uniform magnetic field $\vec{H}$ it becomes magnetized with a dipole moment $\vec{p}$ given by

$$\vec{p} = 4\pi a^3 \mu_f \beta \vec{H} \quad \text{with} \quad \beta = \frac{\mu_p - \mu_f}{\mu_p + 2\mu_f}, \tag{1}$$

where a is the radius of a spherical particle, μ_p its relative magnetic permeability, and μ_f the relative magnetic permeability of the interstitial fluid. In the following, we consider particles in air, so that $\mu_f = 1$. As long as the magnetic field is constant, the particle experiences no magnetic force. When a second particle is introduced at coordinates (r,α) (Fig. 1), its magnetic dipole interact with that of the first particle which experiences, in dipolar approximation, the force $\vec{F}_{dip}$:

$$\vec{F}_{dip} = 12\pi\mu_0\mu_f a^2 \beta^2 H^2 \vec{f}_{dip},$$

$$\vec{f}_{dip} = \left(\frac{a}{r}\right)^4 [(2\cos^2\alpha - \sin^2\alpha)\vec{e}_r + \sin 2\alpha \vec{e}_\alpha]. \tag{2}$$

This approximation holds as long as the particles are sufficiently far apart or if their relative permeability is close to one so that the secondary field created by the magnetization of one sphere will not significantly affect the magnetization of the other sphere. Even when these conditions are not met, expression (2) shows an important qualitative characteristic of the magnetic interaction force between two particles: it strongly depends on their position with respect to the field direction. For example, when the spheres are aligned in the direction of the field, they attract each other while they repel when they are in the plane perpendicular to the field direction. This is the reason we proposed to use in our experiment a magnetic rotating field to try to avoid this angular dependence.

As we will see in Sec. III, a direct measurement of the interaction force between two spheres shows that the dipolar approximation is not valid in our case, so that we have to use a multipole expansion of the field to calculate the interaction force. Furthermore if the magnetic permeability depends on the magnetic field, the problem is rather complicated. But, in the following section, we will show that, in the magnetic field range we used, μ_p can be regarded as a constant. In this case, the problem of magnetic particles in a magnetic field is analog to the problem of dielectric particles in an electric field. Using a multipole expansion, Klingenberg proposed in 1989 [11] a method to calculate the exact force between dielectric particles in a uniform electric field. He obtained the following expression for the interaction force:

$$\vec{F} = 12\pi\mu_0\mu_f a^2\beta^2 H^2 \vec{f},$$

$$\vec{f} = \left(\frac{a}{r}\right)^4 [(2f_{//}\cos^2\alpha - f_\perp \sin^2\alpha)\vec{e}_r + f_\Gamma \sin 2\alpha \vec{e}_\alpha]. \tag{3}$$

The functions $f_{//}$, $f_\perp$, and f_Γ depend on the distance between the particles and on their magnetic permeability (dielectric permittivity in the case of dielectric particles submitted to an electric field). They are positive and approach one in the limits: $r/a \to \infty$ and $\mu_p/\mu_f \to 1$.

With the method proposed by Klingenberg, it is impossible to study the interaction force between touching spheres for high values of μ_p/μ_f. In 1993, Clercx and Bossis [12] proposed a more efficient method which allows us to calculate the force up to touching spheres for $\mu_p/\mu_f \leqslant 100$. In the following, we will use the results of Clercx and Bossis for the values of the force functions $f_{//}$, $f_\perp$, and f_Γ.

B. The magnetic force exerted on a particle at surface of the heap

In a granular media, when an avalanche takes place, a slip surface appears in the material. If a constant magnetic field is applied to vary the interaction forces between grains, the angle between the slip surface and the direction of the field would play a role. And finally the values of the angle of repose and of the maximum angle of stability should depend on the direction of the field. To avoid this dependence, we

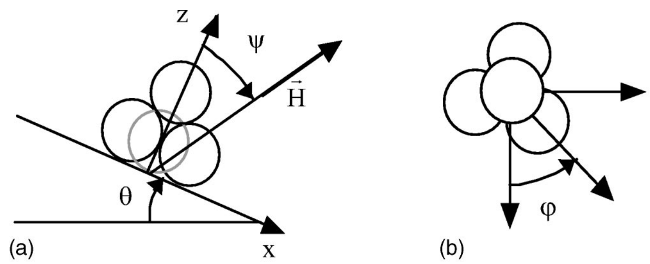

FIG. 2. A particle at the surface of the heap is supposed to be supported by three particles in the layer below. θ is the angle between the surface of the heap and the horizontal plane. The four particles form a regular tetrahedron. The angle Ψ defines the field orientation, and the angle φ defines the orientation of the triangular base.

propose to apply a rotating field in the vertical plane parallel to the quasi-2D box containing the steel beads:

$$\vec{H} = H_0[\sin(\psi)\vec{e}_x + \cos(\psi)\vec{e}_z], \tag{4}$$

ψ being the angle between the normal of the surface of the heap and the direction of the field. We will assume that the avalanche is a superficial model. This hypothesis is supported by visual evidence and, as we will see in Sec. IV, this assumption allows us to explain the main experimental results. Thus, in the following, we will examine the stability of the top layer of particles of the heap interacting with the layer below. Due to the $(a/r)^4$ dependence of the magnetic force, this amounts to studying the interaction between a given particle of the top layer and three adjacent spheres in the layer just below [Fig. 2(a)]. This pyramidal structure is of course a local structure and the orientation of the base triangle defined by φ [Fig. 2(b)] is random. Consequently, to calculate the magnetic force exerted on the top sphere, we have to average the force over all the possible values of φ. The sum of the three interaction forces averaged on φ yields the following expression for the force exerted on the top sphere:

$$\langle\vec{F}\rangle_\varphi = \frac{1}{2}\sqrt{\frac{3}{2}}\pi\mu_0\mu_f a^2\beta^2 {H_0}^2(f_x\vec{e}_x + f_z\vec{e}_z),$$

$$f_x = -(2f_{//} + f_\perp - 4f_\Gamma)\cos\psi\sin\psi, \tag{5}$$

$$f_z = -(1 + 3\cos^2\psi)f_{//} - \tfrac{1}{2}(3\cos^2\psi - 5)f_\perp + (3\cos^2\Psi - 1)f_\Gamma.$$

Furthermore, if the magnetic field is rotating in the plane (x,z), we have to average the force over the possible directions of $\vec{H}$ (i.e., over ψ), and we obtain a force that is normal to the surface of the heap:

$$\langle\vec{F}\rangle_{\varphi,\psi} = \tfrac{1}{4}\sqrt{\tfrac{3}{2}}\pi\mu_0\mu_f a^2\beta^2 {H_0}^2\left(-5f_{//} + \tfrac{7}{2}f_\perp + f_\Gamma\right)\vec{e}_z. \tag{6}$$

At last, it should be noted that the condition to consider only the average force over ψ is that the frequency of the field is sufficiently high for the particles to be at rest during a half period of rotation of the magnetic field. Namely, the half period of the field has to be smaller than the characteristic time for a particle to move on a distance equal to its diameter under gravity:

$$T < 4\sqrt{\frac{a}{g}}. \tag{7}$$

C. The dependence of the MAS on the magnetic interaction force

To link the MAS to the average interaction force, we use a very simple model based on a stability criterion of a single layer of particles lying on the surface of the heap. The choice of such a surface model will be justified by the experimental results presented in Sec. IV. We write the equilibrium of one particle under the contact forces exerted by the particles in the layer below, the magnetic force (6) and the gravity. The contact force is the sum of the component parallel, $\vec{T}$, and normal, $\vec{N}$, to the surface. The projection of the different forces on the axes x and z [Fig. 2(a)] leads to

$$-T + \text{mg}\sin\theta = 0,$$

$$N - \text{mg}\cos\theta - F_{\text{magn}} = 0, \tag{8}$$

where m is the mass of the particle and F_{magn} is taken positive. The measurement of the MAS of the granular material in the absence of a magnetic field, θ_0, allows us to define the internal friction coefficient μ,

$$\mu = \frac{T}{N} = \tan\theta_0, \tag{9}$$

and, in the presence of a magnetic interaction between particles, the MAS of the heap will be given by the following relation:

$$\tan(\theta) = \mu + \frac{\mu F_{\text{magn}}}{\text{mg}\cos(\theta)}, \tag{10}$$

which can also be written as

$$\theta = \theta_0 + a\sin\left(\frac{\mu F_{\text{magn}}}{\text{mg}}\cos(\theta_0)\right) = \theta_0 + a\sin\left(\frac{F_{\text{magn}}}{\text{mg}}\sin(\theta_0)\right). \tag{11}$$

III. EXPERIMENT

A. The particles

We use monodisperse steel ball bearing particles (diameter 1 mm±1%), with density 7.9×10^3 kg m^{-3}. In order to measure the average permeability of the granular material, we have subjected an ensemble of densely packed spheres (ϕ=0.61) to a magnetic field gradient. A long cylindrical tube (length 4 cm, radius 0.45 cm) is filled with the particles and suspended under the tray of a sensitive laboratory balance, in order to measure the magnetic force exerted on the sample (Fig. 3). The magnetic field is supplied by a coil, positioned below the sample. In this configuration, since the

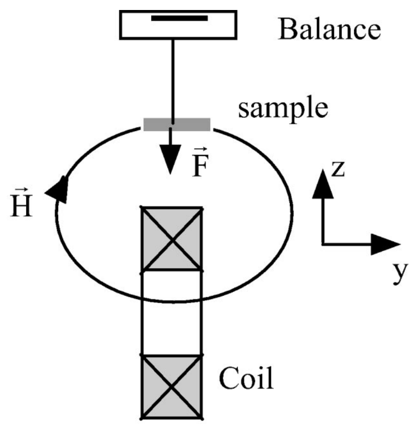

FIG. 3. Measurement of the bulk permeability of the granular material. The magnetic field gradient is normal to the field, and the cylindrical sample is parallel to the field.

long axis of the sample is parallel to the field, the demagnetizing field is low: the demagnetizing factor is evaluated to $N \approx 0.04$ [13]. The magnetic energy may be written as

$$E_{\mathrm{m}} = -\frac{1}{2}\mu_0 V \frac{(\mu_{\mathrm{b}}-1)}{1+N(\mu_{\mathrm{b}}-1)} H_{\mathrm{y}}{}^2, \qquad (12)$$

where μ_{b} is the bulk relative permeability of the sample, V is its volume, and H_{y} is the external magnetic field. The force is thus

$$\vec{F} = -\vec{\nabla} E_{\mathrm{m}} = \mu_0 V \frac{(\mu_{\mathrm{b}}-1)}{1+N(\mu_{\mathrm{b}}-1)} H_{\mathrm{y}} \frac{dH_{\mathrm{y}}}{dz} \vec{e}_{\mathrm{z}}. \qquad (13)$$

Figure 4 shows the variation of the magnetization ($M = \|\vec{F}\| / \mu_0 V dH_{\mathrm{y}}/dz$) with H_{y}. As it can be seen, the granular sample can be considered as a linear magnetic material and the slope of the curve gives the average permeability of the granular material: $\mu_{\mathrm{b}} \approx 6.2$.

The permeability of the steel particles has been estimated by measuring the interaction force between two touching

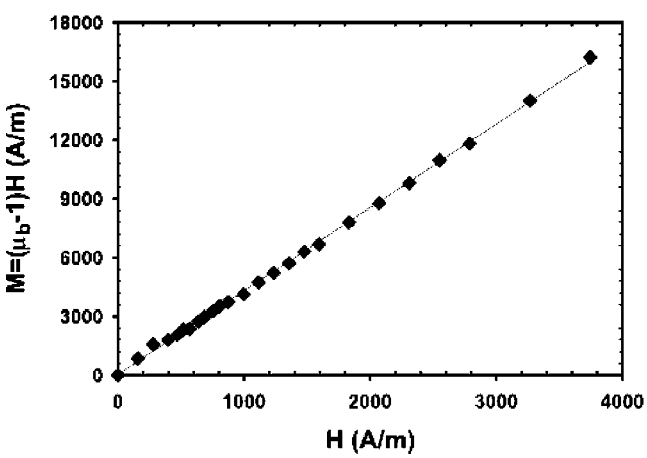

FIG. 4. Measured bulk magnetization vs magnetic field intensity. The granular material can be considered as a linear magnetic material with bulk relative permeability $\mu_{\mathrm{b}} = 6.2$.

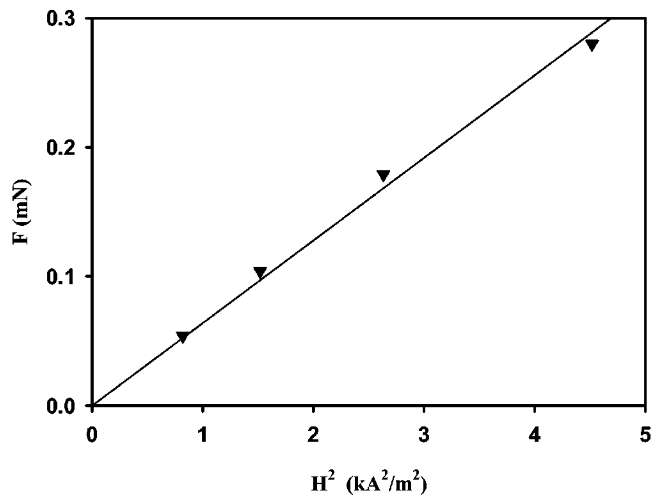

FIG. 5. Magnetic interaction force between two steel spheres (diameter 1 mm) aligned along the field direction vs magnetic intensity squared.

spheres aligned in the direction of a magnetic field. Following Forsyth *et al.* [10], we place two particles in a vertical magnetic field, the top one being fixed, the bottom one holding under the effect of the magnetic interaction. The field is then decreased, until the second particle falls. The weight of the particle gives the magnetic interaction force for this field intensity. To obtain the magnetic force for different field intensities, we glue some additional nonmagnetic masses to the suspended particle. In order to avoid residual magnetization effects, we use an ac magnetic field at the same frequency as in the case of the rotating field (50 Hz). The measured force is displayed versus the square of the field intensity in Fig. 5. A linear regression of the curve gives $F(N) \approx 6.4 \times 10^{-11} H^2 (\mathrm{A}^2/\mathrm{m}^2)$, i.e., from Eq. (3) $\beta^2 f_{//} \approx 43.2$. Volkova [14] gives a polynomial expression for the coefficient $f_{//}$,

$$f_{//} = 0.058 + 0.596\mu_{\mathrm{p}} + 0.015\mu_{\mathrm{p}}{}^2 - (2.78 \times 10^{-5})\mu_{\mathrm{p}}{}^3 \quad \text{for } 10 < \mu_{\mathrm{p}} < 100. \qquad (14)$$

From this expression and the value of the slope of the curve $F(H^2)$, we deduce the permeability of the particles: $\mu_{\mathrm{p}} = 42.1$ and $f_{//} = 49.7$. We have to mention that this determination of the particles permeability is valid only if the magnetic material is linear, i.e., if the permeability does not depend on the magnetic field intensity. This seems to be the case since the force has a quadratic dependence with the field. So in the following, we will use this value for the sphere permeability.

This value of μ_{p} should be compared to the one obtained from the measured mean permeability of the ensemble of packed spheres using an effective medium theory such as Bruggeman's model [15]:

$$\phi \frac{\mu_{\mathrm{p}} - \mu_{\mathrm{b}}}{2\mu_{\mathrm{b}} + \mu_{\mathrm{p}}} + (1-\phi) \frac{1-\mu_{\mathrm{b}}}{2\mu_{\mathrm{b}} + 1} = 0. \qquad (15)$$

This model gives $\mu_{\mathrm{p}} \approx 12.3$. This value is much smaller than the one obtained from the measurement of the interaction force between two spheres. This discrepancy is not surpris-

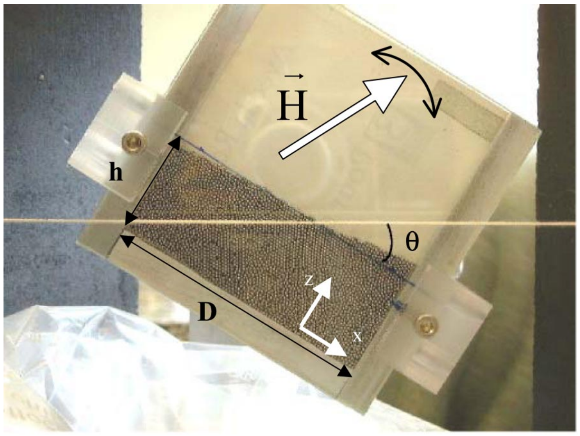

FIG. 6. To measure the MAS, the cell is rotated slowly until the avalanche occurs. The magnetic field is either constant or rotates in the plane (x,z).

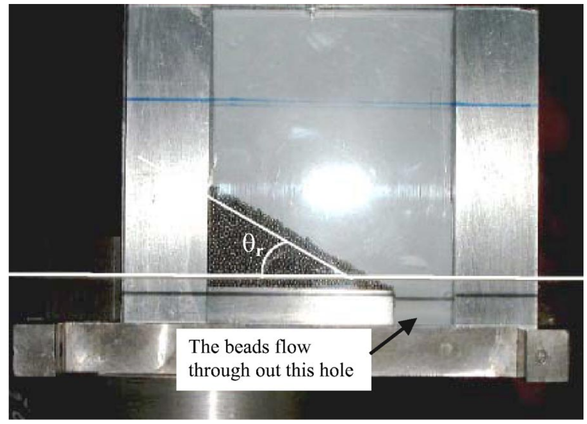

FIG. 7. After the plug has been removed, the grains flow out until the angle of repose is reached. The magnetic field is either constant or rotates in the plane (x,z).

ing since the Bruggeman's model is known to underestimate the value of the permeability of the particles [16]. In the following, we will take the value deduced from the measurement of the magnetic force, $\mu_p \approx 42.1$.

B. Experimental device

A narrow rectangular box is filled with the granular material with a controlled volume fraction $\phi = 0.61$. It is positioned at the center of two perpendicular pairs of coils. The inner and outer diameters of the coils are, respectively, 24 cm and 29 cm and their width is 5 cm. Our work mostly deals with the measurement of the MAS or of the angle of repose when the applied magnetic field is rotating. We also make few measurements in the case of a constant field, to show the importance of using a rotating field, and to compare our results with earlier works described in the literature [10].

To create a rotating magnetic field, the two pairs of coils are run in quadrature phase. The electric current in the coils is supplied by a function generator and amplified by two bipolar amplifiers (Kepco). The resulting magnetic field rotates in the vertical plane perpendicular to the width of the cell containing the beads. Its maximum amplitude is $1430\ \mathrm{A\ m^{-1}}$. Due to the large impedance of each pair of coils, the field frequency is limited to a quite low value, namely, 50 Hz. The value of the characteristic time defined in inequality (7) is $4\sqrt{a/g} \approx 2.8 \times 10^{-2}$ s, which is slightly larger than the period of the field. Finally, the width of the cell is chosen small enough compared to its length and to its height for the demagnetizing field to be much lower than the external field.

We make two different types of measurement, i.e., we measure either the MAS or the angle of repose. In the first case (Fig. 6), we use a box whose width is 5 mm, its length 80 mm, and its height 80 mm. The cell is manually tilted with a low gear. The angular velocity of the cell has been estimated to $6°/\mathrm{min}$. The rotation is stopped when the avalanche occurs and the angular position of the cell gives the angle before the avalanche, i.e., the MAS.

To measure the angle of repose, a fixed cell (width 5 mm, length 60 mm, height 60 mm) with a 15 mm long hole in its bottom is used (Fig. 7). A plug is adjusted in the hole and the cell is filled with the steel particles and placed in the magnetic field. When the plug is removed, the beads start flowing out, until the angle of repose is reached. A picture of the cell is recorded by a digital camera and downloaded to a PC. The surface of the heap is fitted to a straight line. The angle between this line and the horizontal plane gives the angle of repose.

For each value of the magnetic field, six independent measurements were made, yielding a measurement uncertainty of ±1.5°.

IV. RESULTS AND DISCUSSION

A. Constant field

Figure 8 displays the angle of repose versus the square of the magnetic field, in the case of a vertical magnetic field. The best fit shows that the angle of repose scales as $H_0^{1.38}$, i.e., as $F_{\mathrm{magn}}^{0.69}$. In their experiment, Forsyth *et al.* [10] also

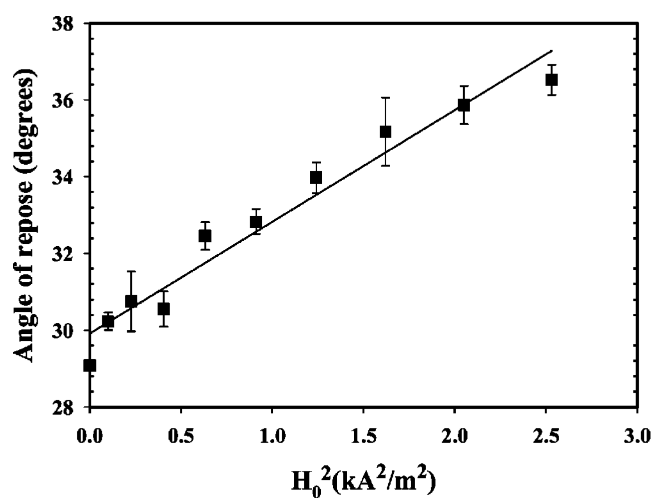

FIG. 8. Angle of repose vs the square of the magnetic field intensity. The magnetic field is vertical and constant. The solid line is a linear regression of the data $\theta_r = 29.9 + 2.91 H^2 (\mathrm{kA^2/m^2})$.

TABLE I. MAS and angle of repose obtained for a constant field $H_0 = 800$ A m^{-1}. The reference axis is the vertical axis. The angle defining the field direction is taken positive clockwise, such as the avalanche angle. So, +45° defines a field roughly normal to the surface of the heap when avalanche occurs and −45° defines a field roughly parallel to the surface.

	Vertical	Horizontal	+45°	−45°
Maximum angle of stability	38.6	38.8	34.4	40.8
Angle of repose	32.3	34	31.3	34.5

apply a vertical magnetic field to the steel beads and they find a linear dependence of the angle of repose on the magnetic force. The main difference between their work and ours is that they apply stronger magnetic fields so that the cohesion is much higher in their experiments than in ours. Nevertheless, the curves they show include few points in the low cohesion regime and a careful look of this part of their results seems to reveal a sublinear dependence of the angle of repose on $H_0{}^2$.

Moreover, we have measured the MAS and the angle of repose obtained for one magnetic field intensity ($H_0 = 800$ A m^{-1}) and various directions of the magnetic field. Table I shows the results. As expected, the values of the angles are strongly dependent on the field direction. This observation emphasizes the importance of applying a rotating field rather than a constant field.

B. Rotating field

Figure 9 displays the MAS together with the angle of repose versus the rotating magnetic field amplitude H_0. The experimental results are fitted by the following expression:

$$\theta - \theta_0 = a \sin(AH_0{}^n). \qquad (16)$$

The best fit, which is shown in Fig. 9 together with the experimental values, gives $A = 1.7 \times 10^{-7}$ and $n_{\mathrm{expt}} = 1.95$. As

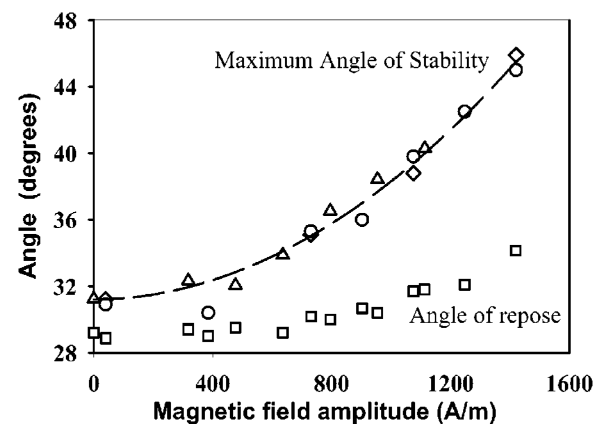

FIG. 9. Measured maximum angle of stability vs rotating magnetic field amplitude for three initial values of the height of the heap. (◇) 2.75 cm; (○) 3.2 cm; (△) 4.4 cm. Dashed line— best fit of the MAS curve to the relation (16); $n_{\mathrm{expt}} = 1.95$; $A_{\mathrm{expt}} = 1.7 \times 10^{-7}$; (■)—angle of repose vs rotating field intensity.

expected from Eq. (11), the MAS variation is quadratic with the magnetic field amplitude.

Our model is of course oversimplified but the qualitative agreement is rather good. Concerning the quantitative comparison, the theoretical coefficient A in Eq. (16) can be calculated using Eqs. (6) and (11). In Eq. (6), $f_{//} = 49.7$ and $f_\perp$ and f_Γ are taken equal to zero since, for high permeability materials, they are much smaller than $f_{//}$ (see, for instance, Refs. [11,14]) and we obtain $A_{\mathrm{th}} = 4.1 \times 10^{-7} (\mathrm{A/m})^{-2}$, whereas the fit of the experimental data gives $A_{\mathrm{expt}} = 1.3 \times 10^{-7} (\mathrm{A/m})^{-2}$. The calculated value of A is three times larger than the measured one. This discrepancy may have different origins. Evaluating the magnetic force, we consider that the force exerted by each particle on the top particle is the same than the force that would exist if the other two base particles were not there. Clercx and Bossis [12] have shown that this assumption is a rather crude approximation. For instance, they obtained that, in a cluster of three particles laid on an equilateral triangle, the two-particle approach led to a strong overestimation (about twice) of the interaction force. The overestimation of A_{th} could also originate in the existence of a demagnetizing field in the pile. This field is rather small since the applied field rotates perpendicularly to the narrow cell width. Nevertheless, it may not be negligible: a rough order of magnitude of the demagnetizing field can be obtained by evaluating the demagnetizing field in an ellipsoid whose polar axis is 5 mm and equatorial axis 80 mm. For such an ellipsoid, the demagnetizing factor in the direction of the polar axis is about 0.9 and so 0.05 in the other two directions. This leads to an internal magnetic field 25% lower than the external applied field.

At last, our model does not take into account the influence of the wall of the cell. Indeed, previous works on dry [18] or immersed [19] granular media without cohesion showed that the maximum angle of stability strongly depends on the ratio of the gap between the walls over the diameter of the beads. It appears that the maximum angle of stability decreases when the gap increases with a characteristic length $B_{\mathrm{m}} \approx 4d$ for glass beads ($d = 1$ mm). Hence wall effects may not be negligible in our case.

We find another noticeable feature of this granular system: we have made several measurements of the MAS for different initial heights of the heap (2.75 cm, 3.2 cm, and 4.4 cm). It clearly appears in Fig. 8 that the MAS do not depend on the height. This observation is the reason we used a surface model to calculate the MAS.

The dependence of the MAS on the height of the pile is still controversial and several models exist that predict different behaviors concerning this precise point. Using the analysis due to Mohr and Coulomb, Halsey and Levine have calculated an upper bound for the MAS [7], which depends on the height h of the granular heap, namely,

$$\tan\,\theta = \mu + \frac{c}{\rho g h \cos\,\theta}, \qquad (17)$$

where μ and c define the criterion for failure in the heap, i.e.,

$$\tau < \mu\sigma + c, \tag{18}$$

where τ is the tangential stress along the slipping plane and σ the normal stress. μ is a phenomenological friction coefficient of the material and c its cohesion.

However, other models exist, which do not predict the same dependence of the MAS on the height of the heap. Albert *et al.* [2] propose a model that takes into account only the surface mechanisms, yielding avalanche angles independent of the heap size. They predict a variation of the MAS linear with the interaction force between grains. Actually, our model is a very simplified version of the model of Albert *et al.* leading to identical qualitative behaviors. In an experimental work dealing with the measurement of the angle of repose in wet granular media, Tegzes *et al.* [1] have shown that in a strong cohesion regime, the angle of repose depends on the height of the heap, whereas in a weak cohesion regime (as in our case), such a dependence does not exist. We believe that in this last regime, due to the weakness of the cohesion, the avalanche is a surface phenomenon, and that, consequently, the angles of avalanche are independent of the height of the pile, as it is the case for noncohesive materials.

In the experiments of Valverde *et al.* [9] on fine granular powders, the avalanche angle depends on the size of the heap, but in this case, the important dimension seems to be the length D of the cell (see Fig. 5) rather than the height. They compare their experimental data to the so-called wedge model [9,17] that they generalize to take into account the cohesion of the material and different shapes of the slipping wedge. They show that this model is appropriate in the high cohesion regime exhibited by the fine powders they use (contrarily to our experimental system where the cohesion is weak). However, such an analysis is difficult to be performed in our case since the predictions of the dependence of the angle versus the cohesion depends on the choice of the wedge shape.

Concerning the angle of repose, the measured angle at zero field is slightly lower than the MAS (2° lower), which is usual for dry granular media. The magnetic interaction leads to an increase of the angle of repose, but the influence of the cohesion induced by the field on the angle of repose is less pronounced than in the case of the MAS. The experimental results seem to indicate that the dependence of the angle of repose on the magnetic field is quadratic but the variation range of the angle is too small for us to be more affirmative. Unfortunately, at present, we are not able to create a larger rotating magnetic field. It should be interesting to make further experiments with higher field intensities using a device such as the one proposed by Martin *et al.* [20]. Another way could be to perform a similar experiment on smaller grains, such as iron powders. The ratio of the magnetic force over weight should be greater in this case, leading to higher cohesion regime.

V. CONCLUSION

In this paper, we have presented some results on the measurement of the maximum angle of stability and of the angle of repose for a cohesive granular material. The cohesion between grains (millimeter steel spheres) is induced by a magnetic field.

Measurement of both the MAS and the angle of repose in static magnetic field exhibits a strong dependence of the values of these angles on the direction of the field. Thus, we emphasized the importance of applying a rotating magnetic field rather than a constant one. This may be especially important for the measurement of the maximum stability angle, since in the case of a static field, the particles are submitted to a variable interaction depending on the varying angular position of the cell before avalanche.

Concerning the experiments made in the presence of a rotating magnetic field, due to the low range of the field intensities, the cohesion is weak (the magnetic interaction force is always lower than the weight of a particle). In this low cohesion regime we have been able to bring out a few characteristics of the behavior of a cohesive granular material. The measured angles have been shown to depend linearly on the interaction force between grains. No variation of the maximum angle of stability of the pile has been noticed when its initial height is changed. A very simple model based on a stability criterion of a single particle lying at the surface of the heap shows a good qualitative agreement with experimental data.

In the presence of a rotating magnetic field, we have also measured the angle of repose. We show that the difference between MAS and angle of repose, which is small for dry granular media, grows up with the interaction between grains.

Finally, we plan to make further measurements in a higher cohesion regime to determine if the dependence of the MAS remains linear with the interaction force between particles and if the independence of the MAS with the height of the pile holds.

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